Exercise 53

Find an equation of the tangent line to the curve at the given point.

$$y = \sin(\sin x), \quad (\pi, 0)$$

Solution

A point on the tangent line is known, so all that's needed is its slope. Take a derivative of the given function

$$y' = \frac{d}{dx}\sin(\sin x) = \cos(\sin x) \cdot \frac{d}{dx}(\sin x) = \cos(\sin x) \cdot \cos x = \cos(\sin x)\cos x$$

and evaluate it at $x = \pi$.

$$y'(\pi) = \cos(\sin \pi) \cos \pi = -1$$

Therefore, the equation of the tangent line to $y = \sin(\sin x)$ at $(\pi, 0)$ is

$$y - 0 = -1(x - \pi).$$

Below is a graph showing the function and the tangent line.

